It might be supposed that relation (14.2) is the equation of minimal coupling and is always satisfied not just in linear approximation of a weak field f_{ni}. A theory with such a coupling equation would then belong to the class of so-called "quasilinear" theories of gravitation (in the terminology of Will). However, as is shown in the work [24], any "quasilinear," asymptotically Lorentz-invariant theory of gravitation contradicts the results of experiments. Therefore, the relation (14.2) must only be the expansion of the minimal coupling equation up to linear terms in a weak field f_{ni}.

Thus, the equation of minimal coupling must be a quadratic equation in the field f_{ni}:

$$g_{ni}=\gamma_{ni}+f_{ni}-\frac{1}{2}\gamma_{ni}f+\frac{1}{4}[b_1 f_{nm}f_i^m+b_2 f_{ni}f+b_3\gamma_{ni}f_{ml}f^{ml}+b_4\gamma_{ni}f^2], \tag{14.3}$$

with parameters of minimal coupling b_1, b_2, b_3, and b_4 which are so far undetermined.

As we shall see below, the condition of coincidence of post-Newtonian expressions for the inertial and gravitational masses of a spherically symmetric body leads to the following relation between the parameters of minimal coupling: $2(b_1 + b_2 + b_3 + b_4) = 1$.

It would be possible to consider also more complex coupling equations which in the weak-field approximation go over into the minimal coupling equation (14.3). However, at present we have no justification for such complication, since the equation of minimal coupling (14.3) describes all gravitational experiments.

We therefore carry out all subsequent considerations on the basis of the equation of minimal coupling (14.3). Here we shall consider the condition of absence of singularities of the metric of the effective Riemannian space–time for finite values of the density of matter at the source of the gravitational field as the basic physical requirement imposing definite restrictions on the values of the parameters of minimal coupling. This assumption excludes the appearance in the field theory of gravitation of objects reminiscent of black holes.

Moreover, we require that there be no paradox of Olbers type in the description of the model of the universe.

It should be noted that because of the equation of minimal coupling (14.3) nondiagonal components of the metric tensor of Riemannian space–time g_{nm} can be nonzero even when the nondiagonal components of the gravitational field f_{nm} are equal to zero.

In order that the nondiagonal components of the tensor g_{nm} vanish when the corresponding nondiagonal components of the gravitational field are equal to zero, it is necessary and sufficient that $b_1 = 0$. In this case we arrive at the equation of simplest minimal coupling (the P–M coupling)

$$g_{nm}=\gamma_{nm}+f_{nm}-\frac{1}{2}\gamma_{nm}f_{li}+\frac{1}{4}[b_2 f_{nm}f+b_3\gamma_{nm}f_{li}+b_4\gamma_{nm}f^2] \tag{14.4}$$

The condition of coincidence of post-Newtonian expressions for the gravitational and inertial mass of a static, spherically symmetric body requires that the parameters of the P–M coupling (14.4) satisfy the relation $2(b_2 + b_3 + b_4) = 1$.

15. Conservation Laws in the Field Theory of Gravitation

Conservation laws valid for all theories of gravitation of class (A) were obtained in Sec. 12. The presence in theories of this class of a differential conservation law for the density of the total symmetric energy–momentum tensor of a system in flat space–time (12.18) makes it possible to obtain a corresponding integral conservation law.

In Cartesian coordinates we have

$$\partial_n[t_g^{ni}+t_M^{ni}]=0. \tag{15.1}$$

Integrating this expression over some volume V for i = 0 and assuming that across the surface bounding this volume there are no flows of matter, we obtain

$$-\frac{\partial}{\partial t}\int dV\,[t_g^{00}+t_M^{00}]=\oint dS_\alpha t_g^{0\alpha}. \tag{15.2}$$

Thus, in the radiation of gravitational waves the energy of the source must change, whereby if the gravitational waves carry positive energy the energy of the source must decrease.

All these conclusions and relations are also valid for a field theory of gravitation which is a concrete representative of theories of class (A). Since symmetric and canonical energy–momentum tensors differ by the divergence of an antisymmetric tensor of third rank, the conservation laws (12.18) and (15.1) also hold for the canonical energy–momentum tensor.

The canonical energy–momentum tensor of the free gravitational field can be obtained as follows. We write out the equality

$$\frac{\partial L_g}{\partial x^n}=\partial_l\left[\frac{\partial L_g}{\partial(\partial_l f_{ml})}\partial_n f_{ml}\right]-\partial_n f_{lm}\partial_l\left[\frac{\partial L_g}{\partial(\partial_l f_{ml})}\right]. \tag{15.3}$$

According to (13.27), the free gravitational field satisfies the equation

$$\partial_l\left[\frac{\partial L_g}{\partial(\partial_l f_{ml})}\right]=\Box f_{ml}=0,$$

and hence expression (15.3) implies that the divergence of canonical energy–momentum tensor of the free gravitational field is equal to zero. From this we obtain

$$\tilde{t}^n_{gi}=-L_g\delta_i^n+\frac{\partial L_g}{\partial(\partial_n f_{ml})}d_i f_{lm}. \tag{15.4}$$

Using the expression for the Lagrangian density of the free gravitational field (13.9), we obtain

$$\tilde{t}^n_{gi}=\frac{1}{64\pi}\left\{-\delta_i^n\left[\partial_s f_{lm}\partial^s f^{ml}-\frac{1}{2}\partial_s f\partial^s f\right]+2\partial_i f_{ml}\partial^n f^{ml}-\partial_i f\partial^n f\right\} \tag{15.5}$$

To obtain the symmetric energy–momentum tensor of the gravitational field t_g^{ni} we must write the Lagrangian of the gravitational field L_g and the expression for f_{ni} in explicitly covariant form. Passing in expression (13.9) from the Cartesian coordinate system to an arbitrary curvilinear system, we obtain

$$L_g=\frac{\sqrt{-\gamma}}{64\pi}\gamma^{ir}\left[\gamma^{nl}\gamma^{ms}-\frac{1}{2}\gamma^{ml}\gamma^{ns}\right]D_i f_{ml}D_r f_{sn}. \tag{15.6}$$

Similarly, from the expression (13.6) we have

$$f_{ni}=\gamma^{ml}\left[D_l D_m\varphi_{ni}-D_i D_l\varphi_{mn}-D_n D_l\varphi_{mi}+D_n D_i\varphi_{ml}+\gamma_{ni}\gamma^{sr}(D_l D_s\varphi_{rm}-D_s D_r\varphi_{ml})\right]. \tag{15.7}$$

To simplify the writing of the following expressions, we also introduce the notation

$$\begin{aligned}\Lambda^{ik}=&-A^{ml}\left[\partial_m\partial_l\varphi^{ik}-\partial^i\partial_l\varphi_m{}^k-\partial^k\partial_l\varphi_m{}^i+\partial^i\partial^k\varphi_{ml}\right]+\\&+\frac{1}{2}fA^{ik}+A_n{}^n\left[f^{ik}-\frac{1}{2}\gamma^{ik}f\right]+\frac{1}{2}\partial_s\{\varphi_n{}^i[-\partial^sA^{kn}+2\partial^nA^{sk}+\\&+2\gamma^{sk}\partial_lA^{ln}-\gamma^{kn}\partial_lA^{ls}-\partial^kA^{sn}+\gamma^{kn}\partial^sA_l{}^l-2\gamma^{sk}\partial^nA_l{}^l]+\\&+\varphi_n{}^s[\partial^iA^{kn}-\partial^nA^{ik}-\gamma^{ik}\partial_lA^{nl}+\gamma^{ik}\partial^nA_l{}^l]+2\gamma^{ks}A^{nr}\partial^i\varphi_{nr}-\\&-A^{sn}\partial^i\varphi_n{}^k-3A^{kn}\partial^i\varphi_n{}^s+2A^{ks}\partial^i\varphi_n{}^n-\gamma^{ik}A^{nm}\partial^s\varphi_{nm}+\\&+3A^{kn}\partial^s\varphi_n{}^i-A^{ik}\partial^s\varphi_n{}^n-2\gamma^{sk}A^{nl}\partial_l\varphi_n{}^i-2A^{sk}\partial_l\varphi^{li}+A^{ns}\partial_n\varphi^{ik}+\\&+\gamma^{ik}A^{nl}\partial_l\varphi_n{}^s+A^{ik}\partial_n\varphi^{ns}+A_l{}^l[2\partial^i\varphi^{ks}-2\gamma^{ks}\partial^i\varphi_n{}^n-2\partial^s\varphi^{ik}+\\&+\gamma^{ik}(\partial^s\varphi_n{}^n-\partial_n\varphi^{ns})+2\gamma^{ks}\partial_n\varphi^{ni}]\}.\end{aligned} \tag{15.8}$$

The symmetric energy–momentum tensor of the gravitational field can be obtained by substituting expressions (15.6) and (15.7) into relation (12.12). In a Cartesian coordinate system we have

$$\begin{aligned}t_g^{ni}=&\frac{1}{64\pi}\left\{-\gamma^{ni}\left[\partial_s f_{ml}\partial^s f^{ml}-\frac{1}{2}\partial_s f\partial^s f\right]+2\partial^i f_{ml}\partial^n f^{ml}-\partial^i f\partial^n f\right\}+\\&+\frac{1}{16\pi}\left\{\partial_l f^{im}\partial^l f_m{}^n-\frac{1}{2}\partial_m f^{ni}\partial^m f\right\}-\frac{1}{32\pi}\partial_l\{f_s{}^l[\partial^i f^{ns}+\partial^n f^{sl}]+\\&+f_s{}^n[\partial^l f^{si}+\partial^i f^{ls}]-f^{ni}\partial^l f-f^{ls}[\partial^i f_s{}^n+\partial^n f_s{}^i]\}-2\Lambda^{(ni)},\end{aligned} \tag{15.9}$$

where, as usual, symmetrization is performed on indices in parentheses:

$$\Lambda^{(ni)}=\frac{1}{2}(\Lambda^{ni}+\Lambda^{in}).$$

The tensor A^{nm} contained in expression (15.8) in this case has the form

$$A^{nm} = -\frac{1}{32\pi}\Box\left[f^{nm} - \frac{1}{2}\gamma^{nm} f\right].$$

Away from matter $\Box f_{nm} = 0$, and hence the expression for t_g^{ni} simplifies considerably:

$$t_g^{ni} = \tilde{t}_g^{ni} + \frac{1}{32\pi}\partial_l\{f_s{}^l[\partial^i f^{ns} + \partial^n f^{is}] - f_s{}^i\partial^n f^{sl} - f_s{}^n\partial^i f^{sl}\}, \qquad (15.10)$$

where $\tilde{t}_g^{ni}$ is the canonical energy–momentum tensor of the free gravitational field (15.5).

We shall show that in the wave zone the symmetric energy–momentum tensor of the gravitational field t_g^{ni} differs from the canonical energy–momentum tensor $\tilde{t}_g^{ni}$ only by nonwave terms decreasing faster than $1/r^2$. Since in the wave zone we have the expansion

$$f_{nm} = \frac{a_{nm}(t-r,\theta,\varphi)}{r} + O\left(\frac{1}{r^2}\right),$$

for any function $F(f_{ml})$ we have

$$\partial_\alpha F = n_\alpha\frac{\partial}{\partial t}F + O\left(\frac{1}{r}F\right),$$

where

$$n_\alpha = \frac{x_\alpha}{r}.$$

Therefore, expression (15.10) can be written in the form

$$t_g^{ni} = \tilde{t}_g^{ni} + \frac{1}{32\pi}\frac{\partial}{\partial t}\{[f^{0l} + n_\alpha f^{l\alpha}][\partial^i f_l{}^n + \partial^n f_l{}^i] - f_s{}^i\partial^n[f^{0s} + n_\alpha f^{\alpha s}] - f_s{}^n\partial^i[f^{0s} + n_\alpha f^{\alpha s}]\} + O\left(\frac{1}{r^3}\right).$$

Denoting differentiation with respect to time by a dot, from the additional conditions (13.28) we have

$$\dot{f}^{0s} + n_\alpha\dot{f}^{\alpha s} = O\left(\frac{1}{r^2}\right). \qquad (15.11)$$

Integrating this expression on time and setting the constants of integration equal to zero, since waves must not have a part not depending on time, we obtain

$$f^{0s} + n_\alpha f^{\alpha s} = O\left(\frac{1}{r^2}\right). \qquad (15.12)$$

From this it follows that in the wave zone the symmetric energy–momentum tensor of the gravitational field differs from the canonical energy–momentum tensor by a nonwave term decreasing faster than $1/r^2$ with increasing r:

$$t_g^{ni} = \tilde{t}_g^{ni} + O\left(\frac{1}{r^3}\right). \qquad (15.13)$$

Therefore, in the wave zone computations carried out using either the symmetric or canonical energy–momentum tensors of the gravitational field give the same result. These tensors are also equivalent in calculating the integral characteristics of gravitational radiation. Indeed, from expression (15.10) we have

$$t_g^{00} = \tilde{t}_g^{00} + \frac{1}{16\pi}\partial_\alpha\{f^{\alpha m}\dot{f}_m{}^0 - f_m{}^0\dot{f}^{\alpha m}\}.$$

Therefore,

$$\int t_g^{00}dV = \int\tilde{t}_g^{00}dV + \frac{1}{16\pi}\int dS_\alpha[f^{\alpha m}\dot{f}_m{}^0 - f_m{}^0\dot{f}^{\alpha m}].$$

If the boundary of the region of integration is located in the wave zone, then by relations (15.11) and (15.12) we have

$$f^{\alpha m}\dot{f}_m{}^0 - f_m{}^0\dot{f}^{\alpha m} = n_\beta[f^{\alpha m}\dot{f}_m^\beta - \dot{f}^{\alpha m}f_m^\beta] + O\left(\frac{1}{r^3}\right). \qquad (15.14)$$

Choosing as a surface of integration a sphere of radius r ($dS_\alpha = -r^2 n_\alpha d\Omega$) and noting that the right side of expression (15.14) is antisymmetric in the indices α and β, we obtain

$$\int dV t_g^{00} = \int dV \tilde{t}_g^{00} + O\left(\frac{1}{r}\right). \qquad (15.15)$$

Moreover, from relation (15.13) it follows that

$$\int t_g^{0\alpha} dS_\alpha = \int \tilde{t}_g^{0\alpha} dS_\alpha + O\left(\frac{1}{r}\right). \qquad (15.16)$$

Thus, equivalence of the canonical and symmetric energy—momentum tensors in calculating integral characteristics of gravitational radiation is obvious from expressions (15.15) and (15.16).

As will be shown in Sec. 24, the components $\tilde{t}_g^{00}$ and $\tilde{t}_g^{0\alpha}$ are quantities of positive sign, and only the transverse components of a gravitational wave contribute to the energy—momentum. Therefore, because of expression (15.2), in radiating waves the energy of the source is reduced.

To obtain the density of the symmetric energy—momentum tensor of matter in flat space—time t_M^{ni} we note that the metric tensor γ_{ni} enters the Lagrangian density of matter only through the metric tensor of Riemannian space—time. Therefore, the density of the tensor t_M^{ni} can be written in the form

$$t_M^{ni} = T^{nl}\left[1 - \frac{1}{2} f + \frac{b_3}{4} f_{ml} f^{ml} + \frac{b_4}{2} f^2\right] + \frac{1}{2} f^{ni} T^{ml} \gamma_{ml} - \qquad (15.17)$$

$$- \frac{1}{4}\left[b_1 T^{ml} f_m{}^n f_l{}^i + b_2 T^{ml} f_{ml} f^{ni} + 2b_3 f_s{}^i f^{ns} T^{ml} \gamma_{ml} + 2b_4 f^{ni} f T^{ml} \gamma_{ml}\right] - 2\Lambda^{(ni)}.$$

We obtain the expression for Λ^{ni} from formula (15.8) if we set

$$A^{ml} = -\frac{1}{2} T^{ml} + \frac{1}{4} \gamma^{ml} T^{ni} \gamma_{ni} - \frac{1}{8}\left[b_1 T^{nl} f_n{}^m + b_1 T^{nm} f_n{}^l + \right. \qquad (15.18)$$

$$\left. + b_2 \gamma^{ml} T^{ni} f_{ni} + b_2 T^{ml} f + 2b_3 f^{ml} T^{ni} \gamma_{ni} + 2b_4 \gamma^{ml} f T^{ni} \gamma_{ni}\right].$$

It thus follows from the results of this chapter that the gravitational field in the field theory of gravitation with minimal coupling is a field in the spirit of Faraday—Maxwell with the usual properties of a carrier of energy—momentum possessed by other physical fields.

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CHAPTER 3

DESCRIPTION OF GRAVITATIONAL EFFECTS IN THE FIELD THEORY OF GRAVITAITON

16. Post-Newtonian Approximation of the Field Theory of Gravitation

The first question which any theory of gravitation should answer is the question of the correspondence between its predictions and the results of available gravitational experiment.

Until recently the requirements on possible theories of gravitation reduced to the necessity of obtaining Newton's law of gravitation in the weak-field limit and also the description of the three effects accessible to observation: the gravitational red shift in the field of the sun, the curving of a light ray passing near the sun, and the displacement of the perihelion of Mercury.

Thus, the available requirements on possible theories of gravitation were clearly insufficient, since a large number of theories satisfied them. Formulation of qualitatively new experiments was required for further choice of gravitational theories.

At the present time, in connection with the development of experimental technology, primarily cosmonautics, and the increase in the accuracy of measurements, new possibilities have appeared regarding more precise measurement of the orbital parameters of planets (primarily the moon), measurement of the retardation of radio signals in the gravitational field of the sun, and performance of new experiments within the solar system. These experiments make it possible to further restrict the circle of viable theories of gravitation.

Nordtvedt and Will [36] developed a formalism, called the parametrized post-Newtonian formalism to facilitate comparison of results of experiments performed within the solar system with predictions of various metric theories of gravitation (i.e., theories of gravitation according to which the action of a weak gravitational field on all physical process except gravitational processes is realized by a metric tensor of Riemannian space–time).

In this formalism the metric of Riemannian space–time created by some body consisting of an ideal fluid is written as the sum of all possible generalized gravitational potentials with arbitrary coefficients called post-Newtonian parameters. Using these parameters of Will–Nordtvedt, the metric of Riemannian space–time can be written in the form

$$\begin{aligned} g_{00} = 1 - 2U + 2\beta U^2 - (2\gamma + 2 + \alpha_3 + \xi_1)\Phi_1 + \xi_1 A + \\ + 2\xi_w \Phi_w - 2[(3\gamma + 1 - 2\beta + \xi_2)\Phi_2 + (1 + \xi_3)\Phi_3 + 3(\gamma + \xi_4)\Phi_4] + \\ + (\alpha_1 - \alpha_2 - \alpha_3) w^2 U + \alpha_2 w^\nu w^\rho U_{\beta\nu} - (2\alpha_3 - \alpha_1) w^\nu V_\nu; \end{aligned} \quad (16.1)$$

$$g_{0\alpha} = \frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \xi_1) V_\alpha + \frac{1}{2}(1 + \alpha_2 - \xi_1) W_\alpha - \frac{1}{2}(\alpha_1 - 2\alpha_2) w_\alpha U + \alpha_2 w^\beta U_{\alpha\beta};$$

$$g_{\alpha\beta} = \gamma_{\alpha\beta}[1 + 2\gamma U],$$